1.2 Use Segments and Congruence

Before
You learned about points, lines, and planes.

Now
You will use segment postulates to identify congruent segments.

Why?
So you can calculate flight distances, as in Ex. 33.

Key Vocabulary
- postulate, axiom
- coordinate
- distance
- between
- congruent segments

In Geometry, a rule that is accepted without proof is called a **postulate** or **axiom**. A rule that can be proved is called a **theorem**, as you will see later. Postulate 1 shows how to find the distance between two points on a line.

**POSTULATE**

**Postulate 1  Ruler Postulate**

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.

The **distance** between points $A$ and $B$, written as $AB$, is the absolute value of the difference of the coordinates of $A$ and $B$.

In the diagrams above, the small numbers in the coordinates $x_1$ and $x_2$ are called **subscripts**. The coordinates are read as “$x$ sub one” and “$x$ sub two.” The distance between points $A$ and $B$, or $AB$, is also called the **length** of $AB$.

**Example 1  Apply the Ruler Postulate**

Measure the length of $ST$ to the nearest tenth of a centimeter.

**Solution**

Align one mark of a metric ruler with $S$. Then estimate the coordinate of $T$. For example, if you align $S$ with 2, $T$ appears to align with 5.4.

$ST = |5.4 - 2| = 3.4 \ \text{Use Ruler Postulate.}$

The length of $ST$ is about 3.4 centimeters.
**Example 2**  
Apply the Segment Addition Postulate

**Maps**  
The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to St. Louis, Missouri.

**Solution**  
Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate.

\[ LS = LT + TS = 380 + 360 = 740 \]

> The distance from Lubbock to St. Louis is about 740 miles.

**Guided Practice** for Examples 1 and 2

Use a ruler to measure the length of the segment to the nearest \( \frac{1}{8} \) inch.

1. \[ M \quad N \]

2. \[ P \quad Q \]

In Exercises 3 and 4, use the diagram shown.

3. Use the Segment Addition Postulate to find \( XZ \).

4. In the diagram, \( WY = 30 \). Can you use the Segment Addition Postulate to find the distance between points \( W \) and \( Z \)? Explain your reasoning.
1.2  Use Segments and Congruence

**CONGRUENT SEGMENTS** Line segments that have the same length are called **congruent segments.** In the diagram below, you can say “the length of \( AB \) is equal to the length of \( CD \),” or you can say “\( AB \) is congruent to \( CD \).” The symbol \( \equiv \) means “is congruent to.”

**Example 3**  
**Find a length**

Use the diagram to find \( GH \).

**Solution**

Use the Segment Addition Postulate to write an equation. Then solve the equation to find \( GH \).

\[
\begin{align*}
FH &= FG + GH & \text{Segment Addition Postulate} \\
36 &= 21 + GH & \text{Substitute 36 for} \ FH \text{ and 21 for} \ FG. \\
15 &= GH & \text{Subtract 21 from each side.}
\end{align*}
\]

**Example 4**  
**Compare segments for congruence**

Plot \( J(-3, 4), K(2, 4), L(1, 3), \) and \( M(1, -2) \) in a coordinate plane. Then determine whether \( JK \) and \( LM \) are congruent.

**Solution**

To find the length of a horizontal segment, find the absolute value of the difference of the \( x \)-coordinates of the endpoints.

\[
JK = |2 - (-3)| = 5 \quad \text{Use Ruler Postulate.}
\]

To find the length of a vertical segment, find the absolute value of the difference of the \( y \)-coordinates of the endpoints.

\[
LM = |-2 - 3| = 5 \quad \text{Use Ruler Postulate.}
\]

\( JK \) and \( LM \) have the same length. So, \( JK \equiv LM \).

**Guided Practice** for Examples 3 and 4

5. Use the diagram at the right to find \( WX \).

6. Plot the points \( A(-2, 4), B(3, 4), C(0, 2), \) and \( D(0, -2) \) in a coordinate plane. Then determine whether \( AB \) and \( CD \) are congruent.
In Exercises 1 and 2, use the diagram at the right.

1. **VOCABULARY** Explain what $MN$ means and what $PQ$ means.

2. **WRITING** Explain how you can find $PN$ if you know $PQ$ and $QN$. How can you find $PN$ if you know $MP$ and $MN$?

**MEASUREMENT** Measure the length of the segment to the nearest tenth of a centimeter.

3. $A$ $B$

4. $C$ $D$

5. $E$ $F$

**SEGMENT ADDITION POSTULATE** Find the indicated length.

6. Find $MP$.

7. Find $RT$.

8. Find $UW$.


10. Find $BC$.

11. Find $DE$.

**ERROR ANALYSIS** In the figure at the right, $AC = 14$ and $AB = 9$. Describe and correct the error made in finding $BC$.

**CONGRUENCE** In Exercises 13–15, plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.

13. $A(0, 1), B(4, 1), C(1, 2), D(1, 6)$; $AB$ and $CD$

14. $J(-6, -8), K(-6, 2), L(-2, -4), M(-6, -4)$; $JK$ and $LM$

15. $R(-200, 300), S(200, 300), T(300, -200), U(300, 100)$; $RS$ and $TU$

**ALGEBRA** Use the number line to find the indicated distance.

16. $JK$

17. $JL$

18. $JM$

19. $KM$

20. **SHORT RESPONSE** Use the diagram. Is it possible to use the Segment Addition Postulate to show that $FB > CB$ or that $AC > DB$? Explain.
PROBLEM SOLVING

32. SCIENCE  The photograph shows an insect called a walkingstick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest 1/4 inch. About how much longer is the walkingstick’s abdomen than its thorax?

33. MODEL AIRPLANE  In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane’s position at three different points during its flight.

a. Find the total distance the model airplane flew.

b. The model airplane’s flight lasted nearly 38 hours. Estimate the airplane’s average speed in miles per hour.
34. ★ SHORT RESPONSE The bar graph shows the win-loss record for a lacrosse team over a period of three years.

a. Use the scale to find the length of the yellow bar for each year. What does the length represent?

b. For each year, find the percent of games lost by the team.

c. Explain how you are applying the Segment Addition Postulate when you find information from a stacked bar graph like the one shown.

35. MULTI-STEP PROBLEM A climber uses a rope to descend a vertical cliff. Let \( A \) represent the point where the rope is secured at the top of the cliff, let \( B \) represent the climber’s position, and let \( C \) represent the point where the rope is secured at the bottom of the cliff.

a. Model Draw and label a line segment that represents the situation.

b. Calculate If \( AC \) is 52 feet and \( AB \) is 31 feet, how much farther must the climber descend to reach the bottom of the cliff?

36. CHALLENGE Four cities lie along a straight highway in this order: City A, City B, City C, and City D. The distance from City A to City B is 5 times the distance from City B to City C. The distance from City A to City D is 2 times the distance from City A to City B. Copy and complete the mileage chart.

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Mixed Review

Simplify the expression. Write your answer in simplest radical form. (p. 874)

37. \( \sqrt{45} + 99 \)  
38. \( \sqrt{14} + 36 \)  
39. \( \sqrt{42} + (-2)^2 \)

Solve the equation. (p. 875)

40. \( 4m + 5 = 7 + 6m \)  
41. \( 13 - 4h = 3h - 8 \)  
42. \( 17 + 3x = 18x - 28 \)

Use the diagram to decide whether the statement is true or false. (p. 2)

43. Points \( A, C, E, \) and \( G \) are coplanar.
44. \( \overrightarrow{DE} \) and \( \overrightarrow{AG} \) intersect at point \( E \).
45. \( \overrightarrow{AE} \) and \( \overrightarrow{EG} \) are opposite rays.