3.6 Using Perpendicular and Parallel Lines

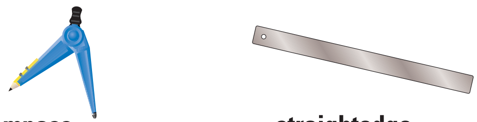
Goal

Construct parallel and perpendicular lines. Use properties of parallel and perpendicular lines.

Key Words

- construction

A **construction** is a geometric drawing that uses a limited set of tools, usually a compass and a straightedge (a ruler without marks).



compass   straightedge

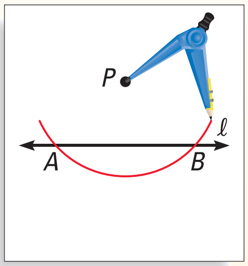
Geo-Activity Constructing a Perpendicular to a Line

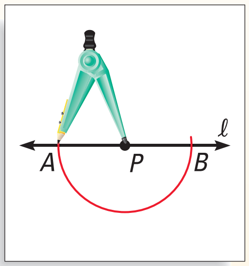
Use the following steps to construct a perpendicular to a line in two different cases:

Line perpendicular to a line through a point *not* on the line.

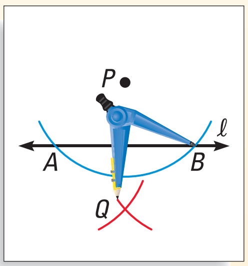
Line perpendicular to a line through a point *on* the line.

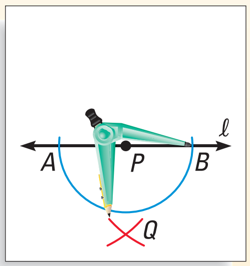
1 Place the compass at point *P* and draw an arc that intersects line ℓ twice. Label the intersections *A* and *B*.



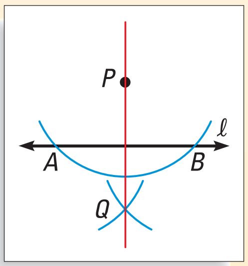


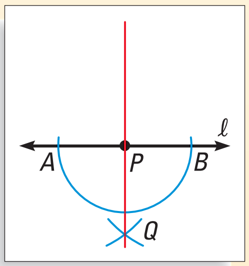
2 Open your compass wider. Draw an arc with center *A*. Using the same radius, draw an arc with center *B*. Label the intersection of the arcs *Q*.





3 Use a straightedge to draw $\overleftrightarrow{PQ}$. $\overleftrightarrow{PQ} \perp \ell$.



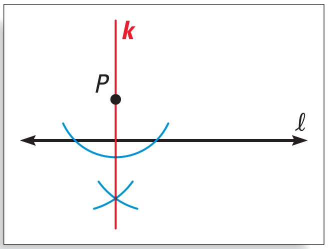


EXAMPLE 1 Construct Parallel Lines

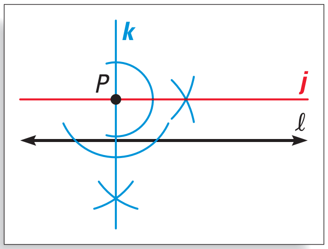
Construct a line that passes through point P and is parallel to line ℓ.

Solution

1 ***Construct*** a line perpendicular to ℓ through P using the construction on the previous page. Label the line k.



2 ***Construct*** a line perpendicular to k through P using the construction on the previous page. Label the line j. Line j is parallel to line ℓ.



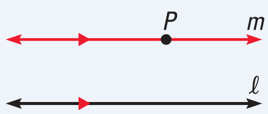
Checkpoint Construct Parallel Lines

1. Draw a line c and a point A not on the line. Construct a line d that passes through point A and is parallel to line c.

POSTULATES 10 and 11

Postulate 10 Parallel Postulate

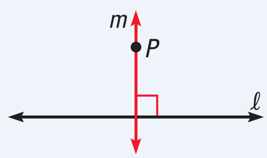
Words If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.



Symbols If P is not on ℓ, then there exists one line m through P such that $m \parallel \ell$.

Postulate 11 Perpendicular Postulate

Words If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

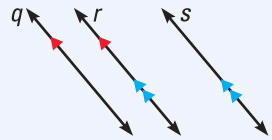


Symbols If P is not on ℓ, then there exists one line m through P such that $m \perp \ell$.

THEOREMS 3.11 and 3.12

Theorem 3.11

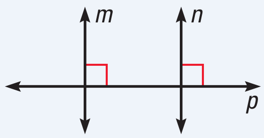
Words If two lines are parallel to the same line, then they are parallel to each other.



Symbols If $q \parallel r$ and $r \parallel s$, then $q \parallel s$.

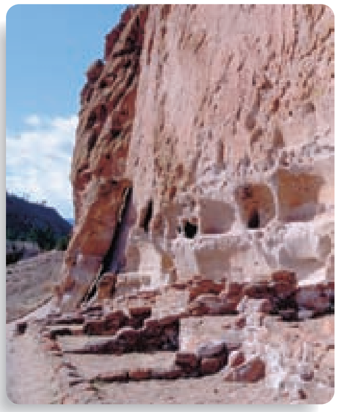
Theorem 3.12

Words In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.



Symbols If $m \perp p$ and $n \perp p$, then $m \parallel n$.

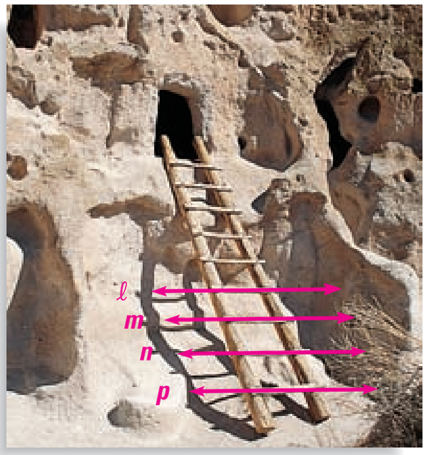
Link to History



CLIFF DWELLINGS were built mostly between 1000 and 1300 by Native Americans. The cliff dwellings above and at the right are preserved at Bandelier National Monument in New Mexico.

EXAMPLE 2 Use Properties of Parallel Lines

Ladders were used to move from level to level of cliff dwellings, as shown at right. Each rung on the ladder is parallel to the rung immediately below it. Explain why $\ell \parallel p$.

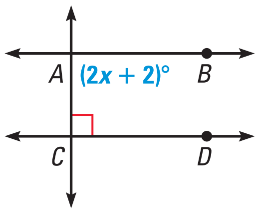


Solution

You are given that $\ell \parallel m$ and $m \parallel n$. By Theorem 3.11, $\ell \parallel n$. Since $\ell \parallel n$ and $n \parallel p$, it follows that $\ell \parallel p$.

EXAMPLE 3 Use Properties of Parallel Lines

Find the value of x that makes $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.



Solution

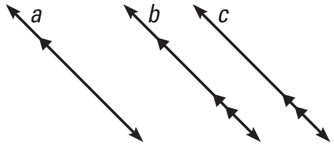
By Theorem 3.12, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ will be parallel if $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are both perpendicular to $\overleftrightarrow{AC}$. For this to be true $\angle BAC$ must measure 90°.

$(2x + 2)^\circ = 90^\circ$	$m\angle BAC$ must be 90°.
$2x = 88$	Subtract 2 from each side.
$x = 44$	Divide each side by 2.

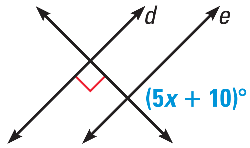
ANSWER ▶ If $x = 44$, then $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

Checkpoint Use Properties of Parallel Lines

2. Use the information in the diagram to explain why $a \parallel c$.



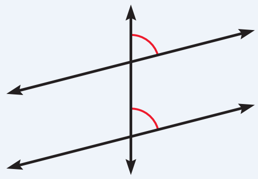
3. Find a value of x so that $d \parallel e$.



You have now studied six ways to show that two lines are parallel.

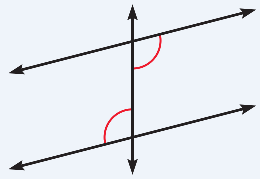
SUMMARY WAYS TO SHOW THAT TWO LINES ARE PARALLEL

Corresponding Angles Converse, p. 137



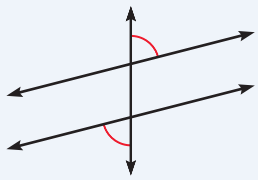
Show that a pair of corresponding angles are congruent.

Alternate Interior Angles Converse, p. 138



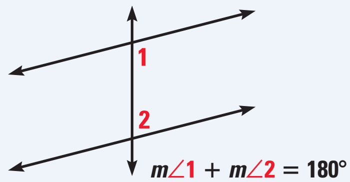
Show that a pair of alternate interior angles are congruent.

Alternate Exterior Angles Converse, p. 138



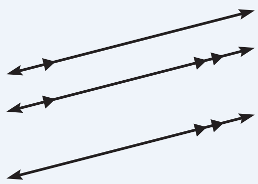
Show that a pair of alternate exterior angles are congruent.

Same-Side Interior Angles Converse, p. 138



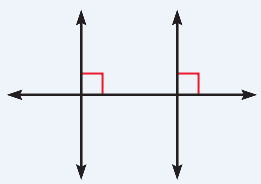
Show that a pair of same-side interior angles are supplementary.

Theorem 3.11, p. 145



Show that both lines are parallel to a third line.

Theorem 3.12, p. 145



In a plane, show that both lines are perpendicular to a third line.

3.6 Exercises

Guided Practice

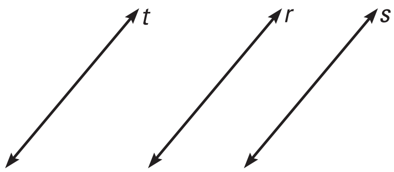
Vocabulary Check

1. What are the two basic tools used for a *construction*?

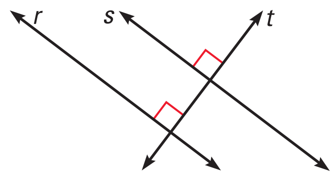
Skill Check

Using the given information, state the theorem that you can use to conclude that $r \parallel s$.

2. $r \parallel t,\ t \parallel s$



3. $r \perp t,\ t \perp s$



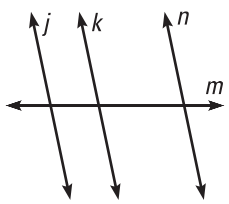
Practice and Applications

Extra Practice

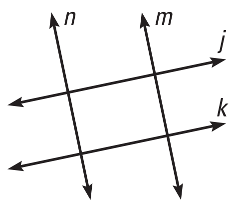
See p. 680.

Logical Reasoning **Using the given information, state the postulate or theorem that allows you to conclude that $j \parallel k$.**

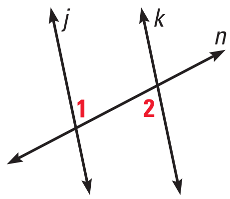
4. $j \parallel n,\ k \parallel n$



5. $j \perp n,\ k \perp n$

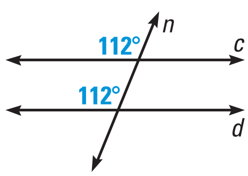


6. $\angle 1 \cong \angle 2$

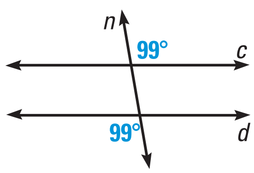


Showing Lines are Parallel **Explain how you would show that $c \parallel d$. State any theorems or postulates that you would use.**

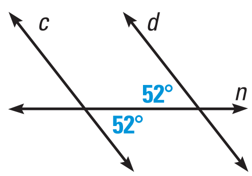
7.



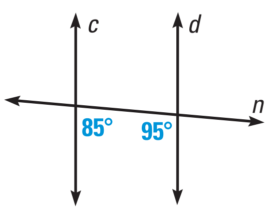
8.



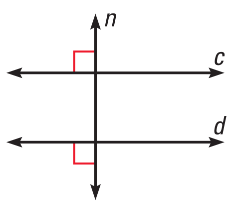
9.



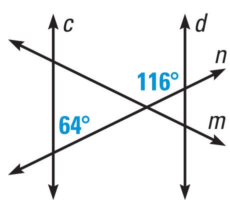
10.



11.



12.



Homework Help

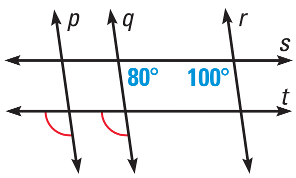
Example 1: Exs. 22–24
Example 2: Exs. 4–12
Example 3: Exs. 19–21

Student Help
CLASSZONE.COM

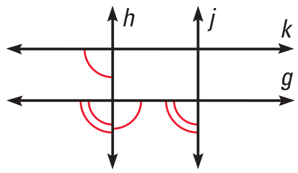
HOMEWORK HELP
Extra help with problem solving in Exs. 13–16 is at classzone.com

Naming Parallel Lines **In Exercises 13–16, determine which lines, if any, must be parallel. Explain your reasoning.**

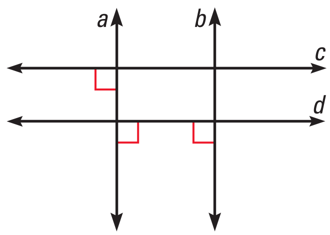
13.



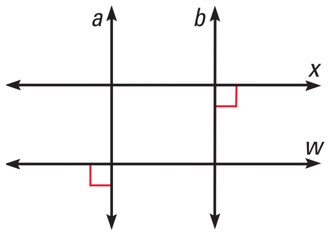
14.



15.



16.



Link to Music

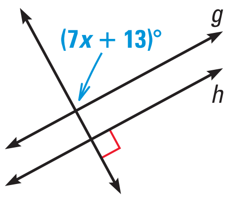
GUITARISTS press their strings against frets to play specific notes. The frets are positioned to make it easy to play scales. The frets are parallel so that the spacing between the frets is the same for all six strings.

17. Guitars In the photo of the guitar at the right, each fret is parallel to the fret beside it. Explain why the 8th fret is parallel to the 10th fret.

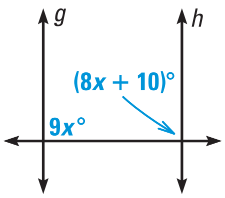
18. Visualize It! Make a diagonal fold on a piece of lined notebook paper. Explain how to use the angles formed to show that the lines on the paper are parallel.

Using Algebra **Find the value of x so that $g \parallel h$.**

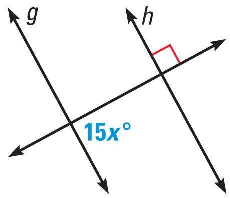
19.



20.



21.



Constructions **In Exercises 22–24, use a compass and a straightedge to construct the lines.**

22. Draw a horizontal line ℓ and choose a point P on line ℓ. Construct a line m perpendicular to line ℓ through point P.

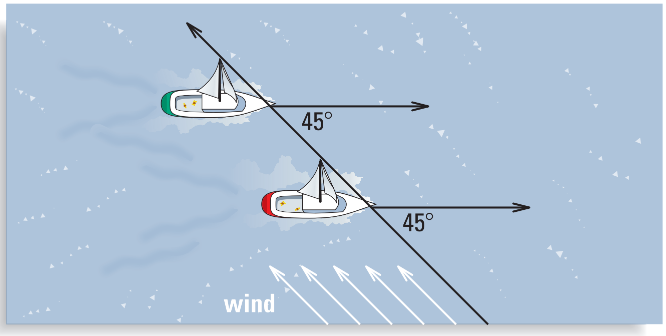
23. Draw a vertical line ℓ and choose a point P to the right of line ℓ. Construct a line m perpendicular to line ℓ through point P.

24. Draw a horizontal line ℓ and choose a point P above line ℓ. Construct a line m parallel to line ℓ through point P.

Student Help

LOOK BACK
For an example of boats sailing at an angle to the wind, see p. 104.

25. Sailing If the wind is constant, will the boats' paths ever cross? Explain.



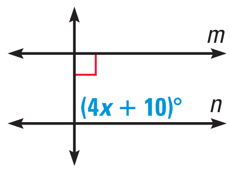
26. Challenge Theorem 3.12 applies only to lines in a plane. Draw a diagram of a three-dimensional example of two lines that are perpendicular to the same line but are not parallel to each other.

Standardized Test Practice

27. Multiple Choice Find the value of x so that $m \parallel n$.

(A) 20 (B) 25

(C) 40 (D) 90

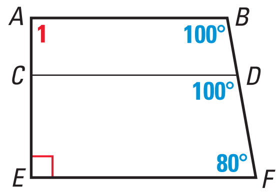


28. Multi-Step Problem Use the information given in the diagram at the right.

a. Explain why $\overline{AB} \parallel \overline{CD}$.

b. Explain why $\overline{CD} \parallel \overline{EF}$.

c. What is $m\angle 1$? How do you know?



Mixed Review

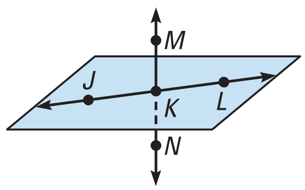
Points, Lines and Planes **Decide whether the statement is *true* or *false*.** *(Lesson 1.3)*

29. N lies on $\overleftrightarrow{MK}$.

30. J, K, and M are collinear.

31. K lies in plane JML.

32. J lies on $\overrightarrow{KL}$.



Plotting Points **Plot the point in a coordinate plane.** *(Skills Review, p. 664)*

33. $A(2, 3)$ **34.** $B(-1, 6)$ **35.** $C(-4, 7)$ **36.** $D(-2, -5)$

Algebra Skills

Expressions **Evaluate the expression.** *(Skills Review, p. 670)*

37. $-5 \cdot 6 - 10 \div 5$ **38.** $-8 + 33 - 14$ **39.** $24 \div (9 + 3)$

40. $4(8 - 3)^2 - 12$ **41.** $48 - 3^2 \cdot 5 - 6^2$ **42.** $[(1 - 8)^2 + 7] \div 8$

Technology Activity

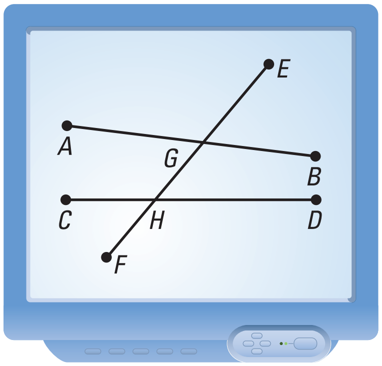
3.6 Parallel Lines and Slope

Question

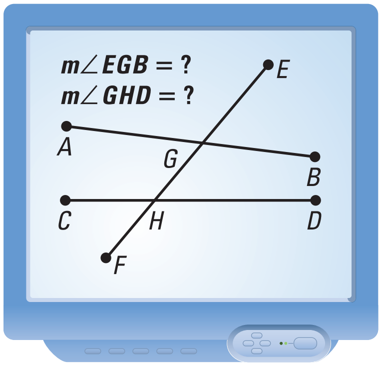
How is slope used to show that two lines are parallel?

Explore

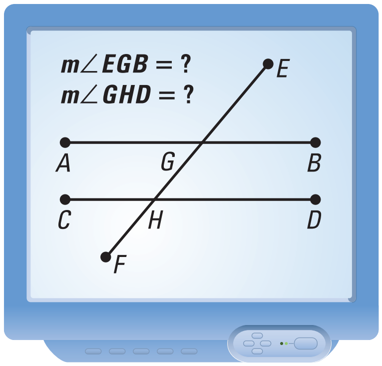
1 Draw and label two segments and a transversal. Label the points of intersection.



2 Measure a pair of corresponding angles.



3 Drag point B until the two angles measured in Step 2 are congruent.



Student Help

SKILLS REVIEW
To review the slope of a line, see p. 665

Think About It

1. Are $\overline{AB}$ and $\overline{CD}$ in Step 3 parallel? What theorem does this illustrate?

In algebra, you learned that the *slope* of a non-vertical line is the ratio of the vertical change (the rise) over the horizontal change (the run). The slope of a line can be positive or negative.

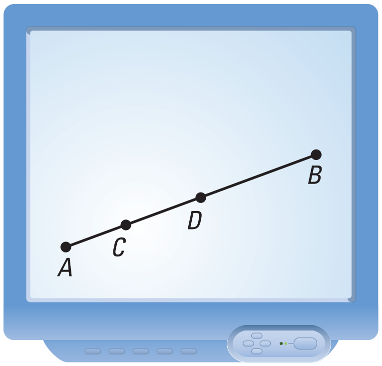
2. Measure the slopes of $\overline{AB}$ and $\overline{CD}$ in Step 3. What do you notice about the slopes?

3. Drag point B to a different position. Drag point D so that the slopes of $\overline{AB}$ and $\overline{CD}$ are equal. What are the measures of the pair of corresponding angles?

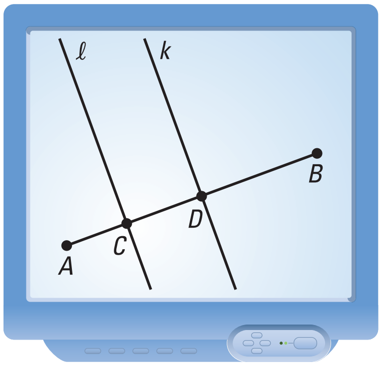
4. Make a conjecture about the slopes of parallel lines.

Explore

4 Draw a non-horizontal segment $\overline{AB}$. Construct and label two points, C and D, on $\overline{AB}$.



5 Construct two lines perpendicular to $\overline{AB}$ through points C and D.



Think About It

5. What theorem allows you to conclude that the lines constructed in Step 5 are parallel?

6. Measure the slopes of the lines constructed in Step 5. Explain how to use the slopes to verify that the lines are parallel.

7. Measure the slope of $\overline{AB}$. Multiply the slope of $\overline{AB}$ by the slope of one of the other lines. What is the result?

8. Drag point B. What happens to the calculation made in Exercise 7 as the slopes of the lines change?

9. **Extension** Construct and label point E on $\overline{AB}$. Construct line m parallel to line k through point E. What theorem allows you to conclude that lines ℓ and m are parallel? Compare the slopes of the lines to verify that they are parallel.